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ENGINEERING MATHEMATICS III

Oct./Nov. 2022

Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

**DIPLOMA IN MECHANICAL ENGINEERING
(PRODUCTION OPTION)
(PLANT OPTION)**

**DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN WELDING AND FABRICATION**

DIPLOMA IN CONSTRUCTION PLANT ENGINEERING

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non-programmable scientific calculator.

This paper consists of EIGHT questions.

Answer FIVE of the following EIGHT questions.

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing.

3. (a) Determine a unit vector \underline{C} perpendicular to vectors $\underline{A} = 4\underline{i} - 3\underline{j} + 2\underline{k}$ and $\underline{B} = 3\underline{i} + 2\underline{j} + 8\underline{k}$. (6 marks)
- (b) Determine the directional derivative of a vector field $F(x, y, z) = x^2yz\underline{i} + xy^2z\underline{j} + xyz^2\underline{k}$ in the direction of the normal to the sphere $Q(x, y, z), x^2 + y^2 + z^2 = 9$ at the point $(2, 1, 2)$. (8 marks)
- (c) Given the vectors $\underline{A} = 2\underline{i} + 3\underline{j} + 4\underline{k}$ and $\underline{B} = 3\underline{i} - 2\underline{j} + 5\underline{k}$; determine the projection of \underline{A} on \underline{B} . (6 marks)

2. (a) Determine the Fourier sine series of the periodic function $f(x) = 4x, 0 \leq x \leq \pi$. (8 marks)
- (b) Determine the Fourier series of the even function defined by

$$f(x) = \begin{cases} 6 + x, & -\pi \leq x \leq 0 \\ 6 - x & 0 \leq x \leq \pi \\ f(x + 2x\pi) \end{cases} \quad (12 \text{ marks})$$

3. (a) Determine the Laplace transform of $f(t) = 3t \cos 8t$ from first principles. (9 marks)
- (b) Use Laplace transforms to solve the differential equation $\frac{d^2x}{dt^2} + 9x = 3 \sin 6t$, given that when $t = 0, x = 1$ and $\frac{dx}{dt} = -4$. (11 marks)

4. (a) (i) Show that $x = 6$ is a solution to the equation

$$\begin{vmatrix} x-1 & -6 & 2 \\ -6 & x-2 & -4 \\ 2 & -4 & x-6 \end{vmatrix} = 0.$$

- (ii) Hence determine the other values of x . (8 marks)

- (b) Use Cramer's rule to solve the equations.

$$\begin{aligned} q_1 + q_2 + 2q_3 &= 4 \\ 2q_1 - q_2 + 3q_3 &= 9 \\ 3q_1 - q_2 - q_3 &= 2 \end{aligned} \quad (12 \text{ marks})$$

5. (a) Solve the differential equation $\frac{dy}{dx} + y \cot x = 5e^{\cos x}$ given that when $x = \frac{\pi}{2}, y = -4$. (5 marks)
- (b) A body of mass 1 kg falls from rest in air where resistance is proportional to the square of the velocity. Determine the velocity of the body at any time t . (15 marks)

6. (a) Given that x_n is an approximation to the root of the equation $x - \cos x = 0$. Using Newton-Raphson method, show that a better approximation is given by:

$$x_{n+1} = \frac{x_n \sin x_n + \cos x_n}{1 + \sin x_n}. \quad (9 \text{ marks})$$

- (b) Table I satisfy a polynomial function $f(x)$.

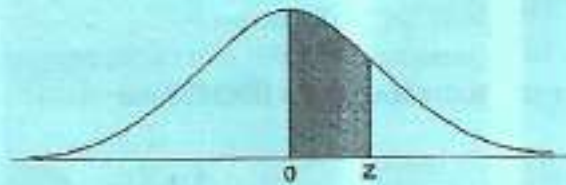
Table I

x	-4	-2	0	2	4	6	8	10
$f(x)$	-135	-13	5	63	305	875	1917	3575

Use a finite difference table and the Newton-Gregory backward difference interpolation to determine $f(4.5)$. (11 marks)

7. (a) Evaluate the double integral $\int_0^{\pi} \int_0^{a(1-\cos\theta)} r^2 \sin\theta dr d\theta$. (8 marks)
- (b) Determine the volume of the solid bounded by the spherical surface $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 + 2y = 0$. (12 marks)

8. (a) Solve the differential equation $(y^2 + 2xy + 1)dx + (2xy + x^2)dy = 0$ given that when $x = 1, y = 1$. (11 marks)
- (b) A point Q moves in a way that its distance from point $R(4, 5)$ is $\frac{3}{4}$ times its distance from the origin. Determine the locus of Q . (9 marks)



Normal Probability

Area under the standard normal curve from 0 to Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063569	0.067495	0.071424	0.075345
0.2	0.079280	0.083166	0.087054	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212280	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267306	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!} t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2 + a^2}\right] = \frac{1}{a} \sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2 + a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n \mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u) du\right] = \frac{1}{s} \mathcal{L}[f(t)]$$

$$\mathcal{L}^{-1}\left[\frac{1}{s} F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)] du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a)$$

$$\mathcal{L}^{-1}[F(s)] = e^{at} \mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \mathcal{L}[f(t)]$$

$$\mathcal{L}^{-1}\left[\frac{d^n F(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as} \mathcal{L}[g(t+a)]$$

$$\mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

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